

# A study of the Bailey–Orowan equation of creep

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A new method was developed to study the Bailey–Orowan equation of creep,  $\dot{\epsilon}_c = r/h$ , where  $\dot{\epsilon}_c$  is the creep rate,  $r$  is the recovery rate and  $h$  is the work-hardening coefficient. The method was to vary the strain rate,  $\dot{\epsilon}$ , around the creep rate,  $\dot{\epsilon}_c$ , and to measure the corresponding stress rate,  $\dot{\sigma}$ . In a plot of stress rate against strain rate, a straight line was obtained. The slope of the straight line was equal to  $h$ , and the intersection of the straight line with the stress axis was equal to  $-r$ , as in the equation  $\dot{\sigma} = -r + h\dot{\epsilon}$ . The creep test under a constant stress is a special case of this equation when the stress rate,  $\dot{\sigma}$ , is zero. The above measurement was carried out within a very small stress variation, less than 1% of the total stress, so that the values of  $r$  and  $h$  were not disturbed. The creep test was performed on Type 316 stainless steel. The creep rate was shown to be equal to the ratio  $r/h$ , but the value of  $h$  was approximately equal to Young's modulus at the testing temperature, rather than, as is commonly believed, to the work-hardening coefficient.

## 1. Introduction

It was first suggested by Bailey [1] that the creep at elevated temperature is a process in which the work hardening from deformation is continually annealed-out by recovery. Steady-state creep occurs when a balance between work hardening and recovery is reached. A simple mathematical formulation of the recovery theory was proposed by assuming that the stress,  $\sigma$ , is a function of time and strain,  $\epsilon$ :

$$\begin{aligned} d\sigma &= \left(\frac{\partial\sigma}{\partial t}\right)_\epsilon dt + \left(\frac{\partial\sigma}{\partial\epsilon}\right)_t d\epsilon \\ &= -r dt + h d\epsilon, \end{aligned} \quad (1)$$

where  $r$  is the recovery rate and  $h$  is the work-hardening coefficient. Since the stress is constant in a creep test,  $d\sigma = 0$  and Equation 1 becomes

$$d\epsilon/dt = \dot{\epsilon}_c = r/h, \quad (2)$$

where  $\dot{\epsilon}_c$  is the creep rate. This is the Bailey–Orowan [2] equation of creep.

Though this concept of creep is generally accepted, experimental verification has proved elusive. In earlier work [3–6],  $r$  was determined by the stress transient dip test in which a stress reduction,  $\Delta\sigma$ , (about 10% of the total stress)

was applied during the creep test and then the incubation time  $\Delta t$  was measured to determine the recommencement of creep. The value of  $r$  was approximated by the ratio  $\Delta\sigma/\Delta t$ . Similarly, a stress jump,  $\Delta\sigma$ , could be applied during the creep test and the corresponding strain increment,  $\Delta\epsilon$ , was then measured. The value of  $h$  could be approximated by  $\Delta\sigma/\Delta\epsilon$  [4–6]. These experiments generally gave an order-of-magnitude agreement between the value of  $r/h$  and the measured steady-state creep rate. However, this method of measuring  $r$  and  $h$  was seriously questioned by Lloyd and McElroy [7–9], who presented evidence that the observed incubation time in the stress dip test was a consequence of anelasticity. Anelasticity could also not be ruled out in the stress–jump test. The measurements of  $r$  and  $h$  are therefore dubious.

In this work a new method has been developed to study the Bailey–Orowan equation. This method is based on the following equation derived from Equation 1 by dividing each term by  $dt$ ,

$$\dot{\sigma} = -r + h\dot{\epsilon}, \quad (3)$$

where  $\dot{\sigma}$  is the stress rate  $d\sigma/dt$  and  $\dot{\epsilon}$  is the strain rate  $d\epsilon/dt$ . Since creep tests are conducted under constant stress, the Bailey–Orowan equation is

only a special case of Equation 3 when the stress rate  $\dot{\sigma}$  is zero and the strain rate,  $\dot{\epsilon}$ , is equal to the creep rate,  $\dot{\epsilon}_c$ . Experimentally, the strain rate  $\dot{\epsilon}$  was varied around the creep rate,  $\dot{\epsilon}_c$ , and the corresponding stress rate,  $\dot{\sigma}$ , was recorded. A straight line was obtained in a plot of stress rate against strain rate and, according to Equation 3, the slope of the straight line should be equal to  $h$  and the intersection of the straight line with the stress axis should be equal to  $-r$ . The measurements of the stress rate and the strain rate variations were taken within a very small stress variation, less than 1% of the total stress, so that the values of  $r$  and  $h$  were not disturbed. Besides the verification of Equation 2, the dependence of  $r$  and  $h$  on creep strain, temperature and stress was also studied in this experiment.

## 2. Experimental procedure

Creep specimens were machined from a 2.54 cm diameter rod of Type 316 stainless steel. The composition of the steel is 0.06 wt% C, 1.57 wt% Mn, 0.029 wt% P, 0.023 wt% S, 0.75 wt% Si, 16.8 wt% Cr, 10.7 wt% Ni and 2.16 wt% Mo. The specimen geometry is shown in Fig. 1. All specimens were annealed in air at 1050°C for 30 min and then air-cooled with a resultant average grain size of 60  $\mu$ m. Specimens were creep tested in a mechanically driven model 1125 Instron Universal Testing Machine. This machine is ideal for this experiment because of a special "hold" control designed to hold the specimen at a constant load. A load limit can be selected by a ten-turn potentiometer in the range of the load cell. When the increasing load reaches the load limit during the cross-head upward movement, the hold control stops the cross-head and allows the specimen to relax under a fixed cross-head. A less than 1% decrease of the total load reactivates the "up" control, and the cross-head starts moving up again

at the selected speed. The load is again increased until it reaches the load limit and triggers the hold control. By repeating this process the sample is subjected to a stepped creep test with a cyclic load amplitude of less than 1% of the total stress. To begin a test the cross-head was first brought up at high speed until the load limit was reached. Then it was changed to low speed, generally  $8.33 \times 10^{-7} \text{ m sec}^{-1}$  to complete the creep test. This stepped creep test is actually equivalent to a real creep test conducted under constant load. The analysis will be shown later.

We shall denote the strain rate of the specimen in the loading period by  $\dot{\epsilon}_1$ , and that in the holding period by  $\dot{\epsilon}_h$ . The corresponding stress rates are denoted by  $\dot{\sigma}_1$  and  $\dot{\sigma}_h$ , respectively. Obviously  $\dot{\sigma}_1$  must be positive while  $\dot{\sigma}_h$  is negative because the load increases in the loading period but decreases in the holding period. The strain rate of the specimen in the loading period,  $\dot{\epsilon}_1$ , is directly controlled by the cross-head speed. So different strain rates  $\dot{\epsilon}_1$  can be obtained simply by choosing different cross-head speeds. The cross-head speed used for normal creep tests is  $8.33 \times 10^{-7} \text{ m sec}^{-1}$ , but when  $r$  and  $h$  are to be measured the cross-head speed is varied over a wide range of speeds to obtain different strain rates,  $\dot{\epsilon}_1$ , and stress rates,  $\dot{\sigma}_1$ . This kind of measurement can be performed at any stage of creep. The variable cross-head speed unit allows the choice of any other cross-head speeds, in addition to those controlled by standard push buttons.

Creep tests were conducted in air at 750°C under a constant load limit with an initial stress of 103.4 MPa. The temperature was maintained within  $\pm 2^\circ \text{C}$  throughout the length of the specimen by a seven-zone furnace with a temperature controller. The extension of the specimen was measured by means of an extensometer attached to the shoulders of the specimen. An Instron strain

SHOULDERS FOR EXTENSOMETER ATTACHMENT

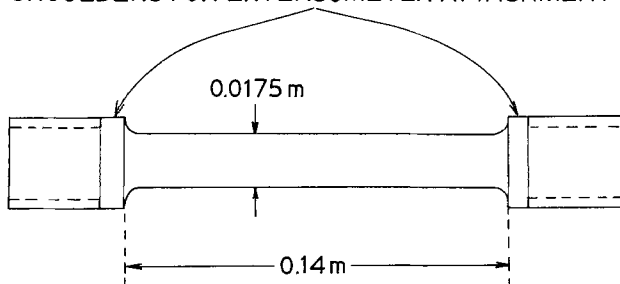


Figure 1 Configuration of the cylindrical specimen.

gauge with a 2.54 cm gauge length and a 100% range was used during the normal creep test to measure the average elongation rate. But this strain gauge was not sufficiently sensitive to measure the small elongation rate variations required to calculate  $r$  and  $h$ . Another high-sensitivity strain gauge with a 1.27 cm gauge length and a 10% range was used with satisfactory results.

The average elongation of the specimen as a function of creep time was recorded with a strip chart recorder fitted to the Instron machine. Since the stepped motion of the elongation was too small to show up in this recording, the creep curve was smooth and the creep rate could be calculated directly by measuring the slope. However, when the strain rates,  $\dot{\epsilon}_1$  and  $\dot{\epsilon}_h$ , and the stress rates,  $\dot{\sigma}_1$  and  $\dot{\sigma}_h$ , were to be recorded, a high-sensitivity Honeywell 196 strip chart recorder was used because of the digitally adjustable zero and the span units.

The digitally adjustable zero unit was used to suppress the static signal to bring the small changing signals into the range of the span unit which could also be adjusted for proper sensitivity.

An example of the recording is shown in Fig. 2. It is clear that the load and the elongation curves in the loading and the holding periods can be well approximated by a straight line. This means a single strain rate and a single stress rate in each period. It should be noted that during the holding period when the cross-head is stopped the specimen still elongates at a considerable rate. The strain rate and the stress rate in each period were calculated directly from the slopes of the recorded curves. Both were averaged over at least five data points and the scatter was generally within 5% of the average value.

The load amplitude, within which the stress and the strain rates were measured, could also be read

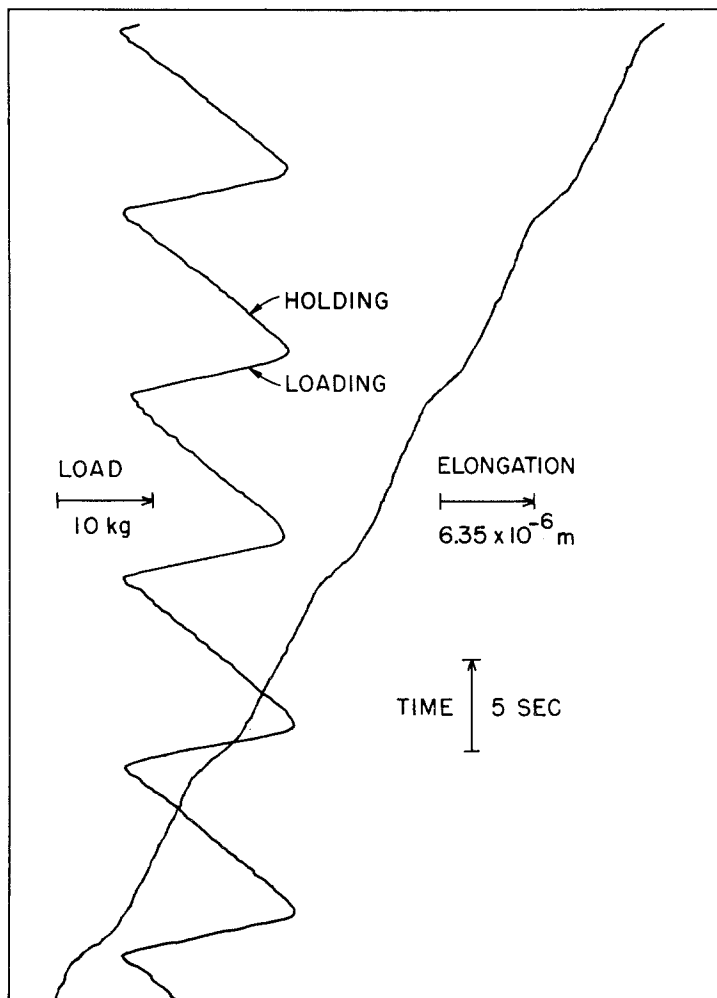


Figure 2 Recorded curves of the load and the extension of the specimens as a function of time.

from the same recording. This load amplitude appeared to increase with the cross-head speed. For example, in a test with a load limit of about 2570 kg, the load amplitude at a cross-head speed of  $8.33 \times 10^{-7} \text{ m sec}^{-1}$  was 13 kg, or 0.5% of the total load. However, when the cross-head speed was increased to  $5.0 \times 10^{-6} \text{ m sec}^{-1}$ , the load amplitude became 19 kg or 0.75% of the total load. In most measurements the maximum load amplitude was generally less than 0.75% of the total load. It was always less than 1% of the total load.

True stress and true strain were used in all calculations. The total elongation at any moment was calculated from the creep curve, and the cross-sectional area was calculated by the assumption of constant total volume. The small un-uniformity of the cross-sectional area near the specimen shoulders was neglected.

### 3. Experimental results

The experimental result for one test is shown in Table I. The first column in the table gives the values of the cross-head speeds,  $\nu$ , which were used to obtain different strain rates and stress rates in loading and in holding,  $\dot{\epsilon}_l$ ,  $\dot{\epsilon}_h$ ,  $\dot{\sigma}_l$  and  $\dot{\sigma}_h$ . The strain rate and stress rate in loading,  $\dot{\epsilon}_l$  and  $\dot{\sigma}_l$ , are shown to increase with the cross-head speed, but in the holding time both the strain rate,  $\dot{\epsilon}_h$ , and the stress rate,  $\dot{\sigma}_h$ , appear to be relatively constant for all cross-head speeds. These data are plotted in Fig. 3. It is clear that a straight line fits these data points very well. The strain rate, at the intersection of the straight line and the strain rate axis, is the creep rate because it corresponds to a constant stress ( $\dot{\sigma} = 0$ ). The creep rate calculated from the creep curve recorded by the Instron strip chart recorder is  $1.34 \times 10^{-6} \text{ sec}^{-1}$ .

Obviously, the straight line must pass through this creep rate point on the strain-rate axis. According to Equation 3, the slope of the straight line is equal to  $h$  and the stress rate at the intersection of the straight line and the stress-rate axis is equal to  $-r$ . Since Equation 3 has proved to be true, the Bailey-Orowan equation is automatically verified because the creep rate,  $\dot{\epsilon}_c$ , must be equal to  $r/h$  by the same equation. This experimental verification of Equation 3 also confirms that the differential form of Equation 1 is valid. Another example tested at  $800^\circ \text{ C}$  is shown in Fig. 4.

It is interesting to see all the datum points from the holding period fall into a small cluster. Since it is small, this cluster can be used to find the best linear fit and it also gives an indication of the consistency of the data obtained by this method.

In an attempt to obtain more datum points, an unloading test was performed in which the cross-head was allowed to move down at a selected speed instead of being held stationary by the hold control. However, in this operation the leadscrews of the Instron machine were required to reverse their directions of rotation, as was the cross-head, and each reversal was followed by a transient period during which measurements could not be made. The transient period is about 30 sec at a cross-head speed of  $8.3 \times 10^{-7} \text{ m sec}^{-1}$  and is inversely proportional to the cross-head speed. A hydraulic testing machine is probably best suited to perform this kind of test.

Since all the tests were conducted with a constant load limit, a special overshooting test which allowed the load to increase above the load limit by releasing the hold control was performed to see if there was any sudden change in stress and strain rates above the load limit. The tests were performed for several cross-head speeds and the

TABLE I The strain rates and the stress rates in loading and holding ( $\dot{\epsilon}_l$ ,  $\dot{\epsilon}_h$ ,  $\dot{\sigma}_l$ ,  $\dot{\sigma}_h$ ) at different cross-head speeds  $\nu$ . The specimen was crept for 44 h at  $750^\circ \text{ C}$  under a constant load of 2570 kg. The present specimen length is 0.166 m, the present cross-sectional area is  $2.04 \text{ cm}^2$ , and the present creep rate is  $1.34 \times 10^{-6} \text{ sec}^{-1}$

$\nu$ ( $10^{-7} \text{ m sec}^{-1}$ )	$\dot{\epsilon}_l$ ( $10^{-6} \text{ sec}^{-1}$ )	$\dot{\epsilon}_h$ ( $10^{-6} \text{ sec}^{-1}$ )	$\dot{\sigma}_l$ (MPa $\text{sec}^{-1}$ )	$\dot{\sigma}_h$ (MPa $\text{sec}^{-1}$ )
4.17	1.67	0.98	0.028	-0.032
6.25	1.83	1.00	0.057	-0.030
8.33	2.14	1.12	0.090	-0.030
12.5	2.44	1.02	0.152	-0.031
16.67	2.91	1.11	0.219	-0.031
25.0	3.69	1.18	0.349	-0.031
33.33	4.72	1.06	0.464	-0.031
41.67	5.35	1.23	0.570	-0.032
50.0	6.64	1.19	0.687	-0.032

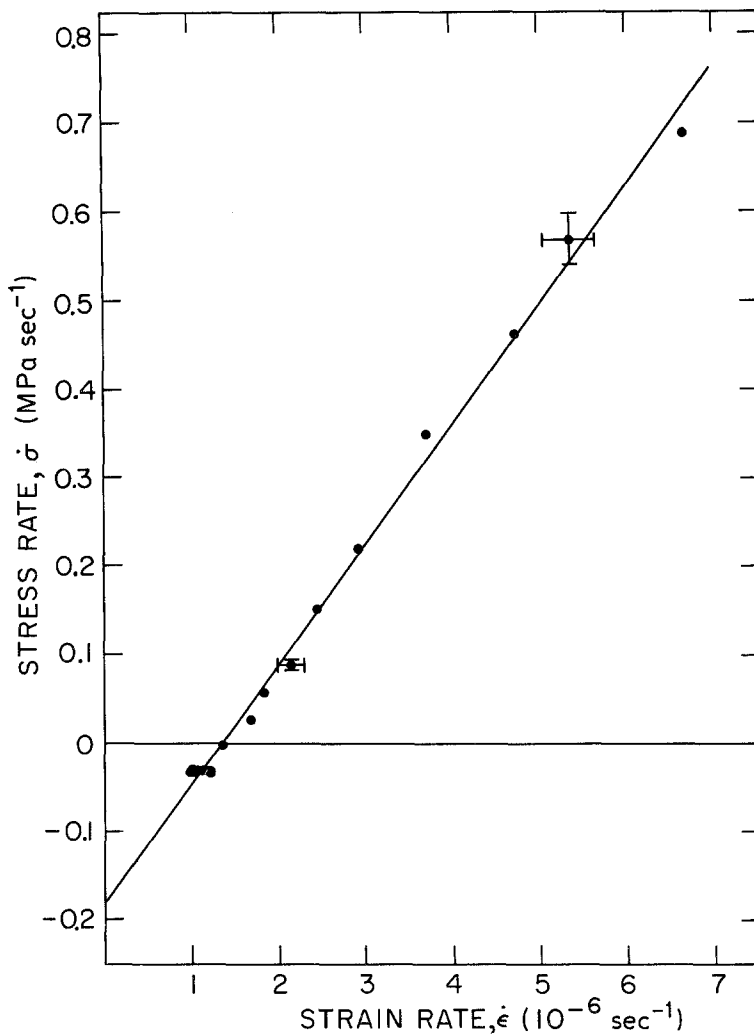


Figure 3 Plot of stress rate,  $\dot{\sigma}$ , against strain rate,  $\dot{\epsilon}$ , for data in Table I. From this figure  $r = 0.18$   $\text{MPa sec}^{-1}$  and  $h = 1.34 \times 10^5 \text{ MPa}$ .

results showed no change for load up to at least 1% above the load limit. An example of the recorded curve is shown in Fig. 5. The strain rate and the stress rate are the same in the overshooting test as those in the normal loading and holding test. In another test the load was allowed to relax below the minimum cyclic load by pushing the stop button in the holding period. No detectable change in the stress and strain rates were observed for load decreased to about 1% below the minimum load.

The possible role of anelasticity was also investigated. Anelasticity has been shown to be relatively constant throughout the creep life of this steel [10]. By complete unloading from the creep load, the anelasticity was measured to be about 15% of the elastic strain. However, in a period of 25 sec after unloading, the anelasticity

that appeared was only about 5% of the elastic strain. Furthermore, in our experiment the load amplitude was applied at a constant rate, rather than a sudden loading or unloading. If the midpoint of the load-time curve is taken as an average, both the time and the load amplitude should be decreased by half. Anelasticity then can contribute only about 1% of the total deformation. Since the time involved in the loading period is generally much less than 25 sec, its effect is even more insignificant. Therefore the effect of anelasticity on the value of the slope,  $h$ , is negligible.

The creep rate was calculated directly from the creep curve recorded by the Instron strip chart recorder, but it can also be calculated by averaging the strain rates  $\dot{\epsilon}_l$  and  $\dot{\epsilon}_h$  over the time of one cycle of loading and holding.

$$\dot{\epsilon}_c = [\dot{\epsilon}_l t_l + \dot{\epsilon}_h t_h] / (t_l + t_h), \quad (4)$$

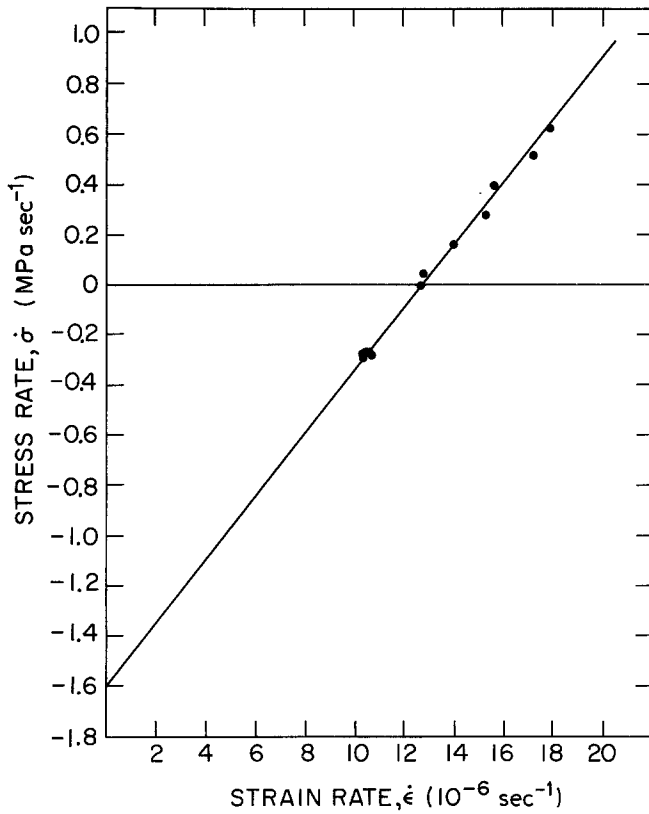


Figure 4 Plot of stress rate against strain rate at 800°C, with  $r = 1.60 \text{ MPa sec}^{-1}$ ,  $h = 1.26 \times 10^5 \text{ MPa}$ .

where  $t_1$  is the loading time and  $t_h$  is the holding time. Another method is to average the distance travelled by the cross-head in the loading time over one cycle of loading and holding

$$\dot{\epsilon}_e = \nu t_1 / (t_1 + t_h) l, \quad (5)$$

where  $\nu$  is the cross-head speed and  $l$  is the specimen length. An example is shown in Table II which is from the same test as that shown in Table I. With a mechanical stop-watch, accurate to 0.2 sec, the loading time  $t_1$  was measured by the signal lights on the control panel of the Instron

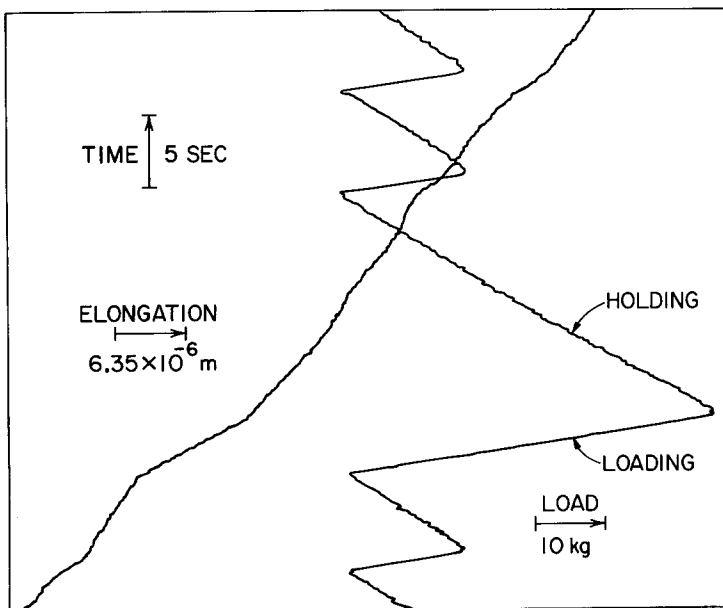


Figure 5 An example of the overshooting test. The specimen elongation rate remains the same when the load is above the load limit.

TABLE II The loading time,  $t_1$ , the holding time,  $t_h$ , and the average elongation rates as a function of cross-head speed. The creep rate is  $1.34 \times 10^{-6} \text{ sec}^{-1}$ . Data were from the same test as Table I

$\nu$ ( $10^{-7} \text{ msec}^{-1}$ )	$t_1$ (sec)	$t_h$ (sec)	$\dot{\epsilon}_c$	
			From Equation 4 ( $10^{-6} \text{ sec}^{-1}$ )	From Equation 5 ( $10^{-6} \text{ sec}^{-1}$ )
4.17	24	19	1.38	1.40
6.25	12.7	21	1.31	1.42
8.33	8.4	22	1.40	1.39
12.5	5.5	23	1.29	1.45
16.67	4.0	24	1.37	1.43
25.0	2.9	27	1.42	1.45
33.33	2.3	29	1.33	1.49
41.67	1.9	29	1.48	1.54
50.0	1.7	30	1.48	1.60

machine. The holding time  $t_h$  was calculated from the recording of Honeywell strip chart recorder. Each  $t_1$  or  $t_h$  is an average of at least five data. The average strain rates calculated from Equations 4 and 5 appear to be relatively constant for a rather wide range of cross-head speeds. Though the average strain rate by Equation 5 appears to be a little high, they both agree with the directly measured creep rate,  $1.34 \times 10^{-6} \text{ sec}^{-1}$ . It is known that, in the special case when the cross-head speed is equal to the elongation rate of the specimen, the load will be a constant and the test becomes a true creep test under constant load. The above analysis shows that if the cross-head speed is not varied significantly from this speed in the loading-holding test the average strain rate is

independent of the cross-head speed. Therefore, the average strain rate must be equal to the creep rate in the special case. This confirms that the stepped creep test in this experiment is equivalent to a true creep test.

It has been shown that in the holding time the specimen still continues to elongate at a considerable rate even though the cross-head is stopped. It is interesting to compare the cross-head speeds,  $\nu$ , with the specimen elongation rates,  $\dot{l}$  ( $\dot{l} = \dot{\epsilon}l$ ). As an example,  $\nu$  and  $\dot{l}$ , calculated from Table I, are plotted in Fig. 6. It is apparent that a straight line can fit these datum points very well. The equation of the straight line is found to be  $\nu = -10 \times 10^{-7} + 5.50\dot{l}$ , where both  $\nu$  and  $\dot{l}$  are in  $\text{m sec}^{-1}$ . Fig. 6 also shows a reference line on which the cross-head

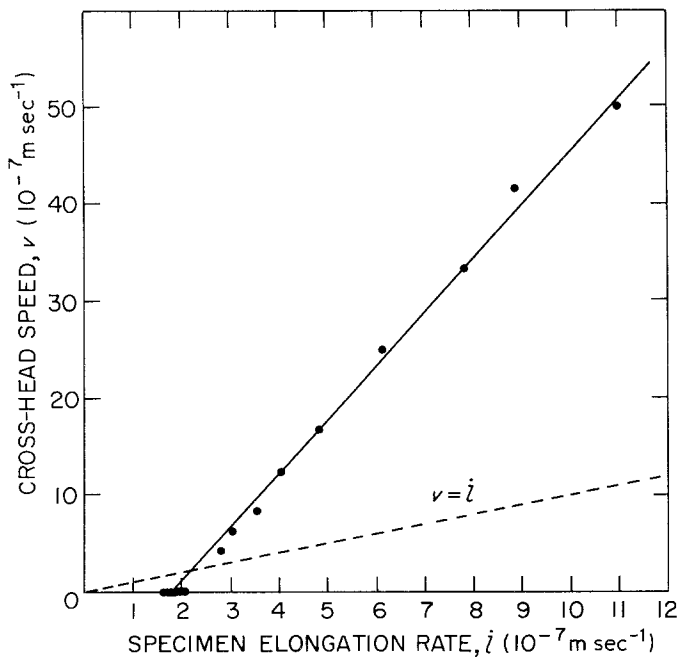


Figure 6 Plot of cross-head speed,  $\nu$ , against specimen elongation rate,  $\dot{l}$ . Data are calculated from Table I.

speed is equal to the specimen elongation rate, for comparison. The intersection between these two lines is the only point at which the cross-head speed is equal to the specimen elongation rate in the present test. As has been mentioned, this corresponds to a constant load condition. The elongation rate at the intersection,  $2.2 \times 10^{-7}$  m sec<sup>-1</sup> or  $1.33 \times 10^{-6}$  sec<sup>-1</sup> in true strain rate, is equal to the directly measured creep rate.

An important point in Fig. 6 is that, generally, there is a large difference between the cross-head speed and the specimen elongation rate. At  $\nu = 6 \times 10^{-7}$  m sec<sup>-1</sup> the cross-head speed is twice the specimen elongation rate, but at  $\nu = 50 \times 10^{-7}$  m sec<sup>-1</sup> the cross-head speed is four times larger. It is obvious that, except when the load is a constant, the cross-head speed should not be used to approximate the specimen elongation rate, especially in the transient period of load change or strain rate change tests.

The slope of the straight line in Fig. 6 is a measure of the composite stiffness of the machine and the connecting rods with respect to the stiffness of the specimen. It decreases as the specimen elongates by creep. For this particular specimen it started at 6.87, decreased to 5.50 with 19% elongation, and reached 3.80 at the end of the test with 41% elongation. It decreases with increasing temperature (5.73 at 700°C, 5.50 at 750°C and 5.26 at 800°C, with  $\sigma = 123$  MPa and approximately 19% elongation), and also decreases with increasing stress (5.50 at 123 MPa and 5.13 at 153 MPa, at 750°C and approximately 19% elongation).

It must be noted that the Bailey–Orowan equation is by no means restricted to steady-state creep. The only requirement for Equation 2 is  $d\sigma = 0$ . This is true at any time of the entire creep life. The values of  $r$  and  $h$  at various stages of creep are listed in Table III. Since it took about half an hour to take enough data to calculate one set of

$r$  and  $h$ , the indicated creep time is the time in the middle of that half hour. The values of  $h$  are generally accurate to  $\pm 0.07 \times 10^5$  MPa. From Table III,  $h$  is approximately a constant from the beginning to the final stage of creep. Furthermore, this constant is equal to the reported Young's modulus of this material at the testing temperature,  $1.39 \times 10^5$  MPa [11]. The three stages of creep can be distinguished by the creep rate. The primary stage of creep, which has a relatively high creep rate, is about one hour. In the secondary stage, lasting for about 30 h the strain rate is not really a constant, but changes very slowly. Then the strain rate starts to increase rapidly and the third stage of creep begins. Just after the data at 44 h were taken, the normal creep process was interrupted to test the effects of temperature and stress. The creep time for the last data in Table III is only an estimate, and the creep rate is larger than would be expected without the interruption.

Since  $h$  is a constant,  $r$  is therefore directly proportional to the creep rate and is larger in the primary stage than in the secondary stage, although it is largest in the tertiary stage of creep.

The effects of temperature and stress on  $r$  and  $h$  were also investigated in this experiment. The tests were conducted on the same specimen as that used in Table III immediately after the data at 44 h were taken. The results are shown in Table IV. The temperature was controlled to within  $\pm 2^\circ$  C throughout the specimen length. The creep rates in Table IV may not be the steady-state creep rates. Temperature is shown to have a large effect on creep rate, but only a minor effect on  $h$ . The reported values of the Young's modulus at each temperature [11] are also listed in Table IV and are seen to be approximately equal to the values of  $h$  at the same temperatures. The increase in stress from 123 to 152 MPa at 750°C increases the creep rate several times but  $h$  still remains the same.

Specimens of 6061-T6 aluminium were also

TABLE III The values of  $r$  and  $h$ , the elongation,  $\Delta l$ , and the creep rate,  $\dot{\epsilon}_c$ , as a function of creep time. The specimen is the same as Table I. The initial length of the specimen was 0.139 m, the initial cross-sectional area was 2.43 cm<sup>2</sup>.

t (h)	$\Delta l$ (%)	$\dot{\epsilon}_c$ ( $10^{-6}$ sec <sup>-1</sup> )	$r$ (MPa sec <sup>-1</sup> )	$h$ ( $10^5$ MPa)
0.4	0.1	1.14	0.15	1.32
1.8	0.5	0.94	0.13	1.38
4.8	1.9	0.87	0.12	1.38
10	4.1	0.86	0.12	1.40
20	7.9	0.90	0.12	1.33
44	19	1.34	0.18	1.34
70	41	5.04	0.70	1.39



TABLE IV Values of  $r$  and  $h$  as functions of temperature and stress. The creep rate,  $\dot{\epsilon}_c$ , and Young's modulus are also listed. The specimen elongation is 19%. These data were measured immediately after the data taken after 44 h, in Table III

Temperature ( $^{\circ}$ C)	Stress (MPa)	$\dot{\epsilon}_c$ ( $10^{-6}$ sec $^{-1}$ )	$r$ (MPa sec $^{-1}$ )	$h$ ( $10^5$ MPa)	$E$ ( $10^5$ MPa)
700	123	0.128	0.019	1.48	1.43
750	123	1.34	0.18	1.34	1.39
800	123	12.7	1.60	1.26	1.33
900	52	1.34	0.16	1.19	1.22
750	153	8.32	1.15	1.38	1.39

tested at 350 $^{\circ}$  C under an initial stress of 15 MPa. The values of  $h$  measured up to 7% elongation are the same value of  $4.8 \times 10^4$  MPa. This is approximately equal to the measured Young's modulus of  $5.0 \times 10^4$  MPa. These values are of course less than the unrelaxed Young's modulus of  $5.5 \times 10^4$  MPa measured in a single crystal at this temperature [12].

#### 4. Discussion

In the previous section the stepped creep test has been used to study the Bailey–Orowan equation of creep. The stepped motion of the cross-head controlled by the loading–holding operation was carried out with a stress amplitude less than 1% of the total stress. In the holding period, it has been shown that the specimen continues to elongate even though the stress is decreasing. If the ratio of the incremental stress,  $\Delta\sigma$ , and the incremental strain,  $\Delta\epsilon$ ,  $\Delta\sigma/\Delta\epsilon$ , in the holding period is calculated, a negative value is obtained. This is contrary to the common observation of the unloading in a tensile test where  $\Delta\sigma/\Delta\epsilon$  is equal to Young's modulus. The explanation is that the amount of unloading in our experiment is very small. If the amount of unloading is increased by moving the cross-head down, the specimen elongation soon stops and the specimen starts to contract. The ratio  $\Delta\sigma/\Delta\epsilon$  will then become Young's modulus too.

In a room-temperature tensile test, at a stress above the yield stress, a small increment in stress,  $\delta\sigma$ , results in a plastic strain increment,  $\delta\epsilon_p$ , much larger than the corresponding elastic strain increment,  $\delta\epsilon_e$ . So the work hardening rate,  $d\sigma/d\epsilon$ , is approximately equal to  $d\sigma/d\epsilon_p$  which is generally orders of magnitude smaller than Young's modulus  $E$  which equals  $d\sigma/d\epsilon_e$ . In this experiment it may have been noticed that the total strain is used in all calculation. The slope  $(\partial\sigma/\partial\epsilon)_t$  is actually equal to  $[\partial\sigma/\partial(\epsilon_e + \epsilon_p)]_t$ . If the plastic strain variation,  $\delta\epsilon_p$ , is much larger than the elastic strain variation,  $\delta\epsilon_e$ , then the value of  $h$  should be orders of magnitude smaller than Young's modulus. The

inclusion of the elastic strain will then contribute only negligible error. However, in this experiment  $h$  is measured to be equal to Young's modulus which is equal to  $(\partial\sigma/\partial\epsilon_e)_t$ . It is apparent that the plastic strain variation,  $\delta\epsilon_p$ , due to the stress cycling must be very small if not zero. This means that the small amplitude stress-cycling affects only the elastic strain and not the creep rate,  $\dot{\epsilon}_c (= d\epsilon_p/dt)$ . In other words, the measured strain rate,  $\dot{\epsilon}$ , is the sum of a varying elastic strain rate,  $\dot{\epsilon}_e$ , and a constant creep rate,  $\dot{\epsilon}_c$ . This also explains why the average strain rate is independent of the cross-head speed in Table II.

It may also be argued that the unloading curve at room temperature also has a slope equal to Young's modulus. In subsequent small-amplitude loading and unloading up to the same load, the stress–strain curve will follow the same path with a slope equal to Young's modulus. This is probably the reason why  $h$  is equal to  $E$  in this experiment. To test this argument the overshooting test was performed. From the room-temperature work-hardening analogy, if the stress is increased above the maximum cyclic load a sudden and large increase in the plastic deformation is expected. The overshooting test was performed for loads up to 1% above the maximum load at several cross-head speeds, but there was no such large increase in the plastic strain observed, and the strain rate remained constant above the maximum load (see Fig. 5). It has been shown that a small load reduction (about 1%) below the minimum load has little effect on the creep rate either. However, in the room-temperature work-hardening, this reduction should be sufficient to stop the plastic flow completely. These results show that the room-temperature work-hardening is very different from the high-temperature creep.

From the above discussions it is found that this experiment cannot be a test of the work-hardening–recovery theory because the value of  $h$  is not that of the work-hardening coefficient. In Equation 1, the recovery rate,  $r$ , is defined as

$(\partial\sigma/\partial t)_\epsilon$ . Mathematically, this means the rate at which the stress is decreased while the total strain is held constant. This is exactly the definition of the stress relaxation rate. The second term  $(\partial\sigma/\partial\epsilon)_t$  has already been identified to be the Young's modulus at the testing temperature. From these evidences the interpretation of Equation 1 as the balance between work-hardening rate and recovery rate is not adequate. Creep is better explained as a process in which the elastic strain is constantly turned into permanent deformation by the stress relaxation. The elastic strain is kept constant by the constant load, which is the driving force of the creep process. The creep rate is equal to the initial stress relaxation rate divided by the Young's modulus.

Though the physical meaning of Equation 1 is shown to be different from the work-hardening–recovery one, it does not mean that theory is wrong. The work-hardening–recovery theory is neither proved nor disproved by this experiment. It only means that the association of Equation 1 with the work-hardening–recovery theory is erroneous. Equation 1 happens to have a different physical meaning and is a different view of the same creep process.

It was pointed out that there is an internal stress,  $\sigma_{\text{int}}$ , in creep and only the effective stress,  $\sigma_{\text{eff}}$ , ( $\sigma_{\text{eff}} = \sigma - \sigma_{\text{int}}$ , where  $\sigma$  is the applied stress) is responsible for creep. The internal stress is measured by the stress-drop test, but that method is only dependable if the anelastic strain for the stress drop,  $\Delta\sigma = \sigma_{\text{eff}}$  is negligible [7]. In estimating the error due to anelasticity it has been shown that there is a considerable amount of anelasticity in Type 316 stainless steel. This means that the internal stress cannot be reliably measured. However, results have shown that only the elastic strain responds to the small-amplitude stress cycling and that the creep rate remains essentially the same. Since the elastic strain is completely determined by the applied stress and Young's modulus, it is above the discussion of the internal stress.

Ahlquist and Nix [13] suggested an alternative expression for the work-hardening–recovery theory by substituting the total stress,  $\sigma$ , in Equation 1 for the internal stress  $\sigma_{\text{int}}$ . The values of the recovery rate and the work hardening rate are then defined, respectively, as  $-(\partial\sigma_{\text{int}}/\partial t)_{\epsilon_p}$  and  $(\partial\sigma_{\text{int}}/\partial\epsilon)_t$  in the new expression. Apparently the present experiment is not capable of verifying this equation because the internal stress variation in

the stress cycling test cannot be measured. However, it has been determined that the plastic strain variation,  $\delta\epsilon_p$ , is much less than the elastic strain variation,  $\delta\epsilon_e$ . Now, if  $h' = (\partial\sigma_{\text{int}}/\partial\epsilon_p)_t$  is equal to the work-hardening rate and is orders of magnitude smaller than  $E$ , the internal stress variation  $\delta\sigma_{\text{int}}$  must be much less than the total stress variation,  $\delta\sigma$ . Since  $\delta\sigma = \delta\sigma_{\text{eff}} + \delta\sigma_{\text{int}}$ , the effective stress variation,  $\delta\sigma_{\text{eff}}$ , must be approximately equal to  $\delta\sigma$ . This is consistent with the assumption in the stress-drop test that, after the stress drop, the effective stress is increased instantly by the same amount but the internal stress first keeps the original value and then starts changing gradually to the new equilibrium value. When the amount of stress drop is equal to the effective stress, the creep rate becomes zero. The specimen will start creeping again only when the internal stress starts changing to the new equilibrium value. It appears that the time period involved in the present experiment must be too short for the internal stress to change appreciably, especially in the loading period. The constant creep rate in the stress-cycling test can be explained such that the effective stress variation,  $\delta\sigma_{\text{eff}}$ , is still very small compared with the total effective stress,  $\sigma_{\text{eff}}$ , so that there is no detectable change in creep rate. From this discussion it appears that the formulation of the work-hardening–recovery theory in terms of the internal stress can be a reasonable one.

There are two technical by-products of the technique developed in this experiment. First, this technique provides a way to measure accurately the initial stress relaxation rate at high temperature. The initial stress relaxation rate cannot be measured by simply stopping the cross-head since the specimen still elongates at a considerable rate the instant the cross-head is stopped. Second, this method can be used to measure the machine stiffness at high temperature. Unless the stiffness is shown to be very large, the cross-head speed cannot safely be used to approximate the specimen elongation rate.

## 5. Conclusion

Equation 1 has been experimentally verified by the stress-rate variation test. The value of  $h = (\partial\sigma/\partial\epsilon)_t$  is found to be the Young's modulus at the testing temperature. The value of  $r = (\partial\sigma/\partial t)_\epsilon$  is actually the stress relaxation rate. So Equation 1 is not a proper formulation for the work-hardening–

recovery theory. The real physical meaning of Equation 1 is that the creep is a process in which the elastic strain is constantly transformed into permanent deformation by the stress relaxation. The creep rate is equal to the initial stress relaxation rate divided by the Young's modulus at the testing temperature.

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